Study of the Nonlinear Duffing Equation Expanded by the Shock-Term to Mitigate Chaotic Components in its Solutions

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Abstract:
The property of the nonlinear Duffing equation is arrival of the chaotic components in its solution. The main objective of this work is to mitigate above components by means of the addition of the shock-term to its right-hand side. In present paper it was found using Runge-Kutta method that negative shock-term indeed essentially mitigates chaos in the solution.

Key Words: Chaos, mitigation, Duffing equation , Runge-Kutta shock-term.

1. Introduction

Consider the known Duffing oscillator governed by the single equation:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + x^3 = K\cos(\omega t) - \delta(t) \quad (1)$$

or by an equivalent set of two first-order non-autonomous equations:

$$\frac{dx}{dt} = y \quad (2)$$

$$\frac{dy}{dt} = -b\frac{dx}{dt} - x^3 + K\cos(\omega t) \quad (3)$$

Here x and y are positions and velocities to be computed for Duffing equation solution, b – damping parameter, x^3 -cubic non-linearity function, K and ω -amplitude and frequency of the appropriate sinusoidal forcing, t -time. These equations are widely used in the study of the properties of the combined chaotic-deterministic oscillations and are the basic models of the theory of deterministic chaos in science and technology. Recognition between the chaotic and deterministic components is of primary interest for many studies such as [1-5] and many others.

2. Establishment of objective

Suppose, that someone wants to develop a robot with an artificial arm [3] that will give to the surgeon tools during surgery. Mathematical modeling in the case under consideration predicted that solution of the Duffing-type equations are both deterministic and chaotic. The question arises: to what extent is the chaos in the movement of the artificial arm acceptable.

It is natural thing to postpone the use of the robot for the period until the means for effective control over chaos are created. In a contrary the objective of this work is supplement the right-hand side of the Duffing equation with the shock-term (mathematically similar to that used in the area of the defibrillation science) that would mitigate the effect of chaos on the solution.
Thus, we consider the generalized Duffing equation of the following form:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + x^3 = K\cos(\omega t) - \delta(t) \quad (4).$$

Negative shock-term defined as

$$\delta(t) = H = \text{constant} \quad (5).$$

for any t changing between the shock initiation and finishing times t_1 and t_2 :

$$t_1 < t < t_2$$

3. Materials and methods

Equations (2)-(3) subjected to the prescribed initial conditions for position x_0 and velocity y_0 at the initial time $t = t_0$ were solved numerically by means of the Runge-Kutta 4^{th} order finite difference method. Present and known phase portraits are depicted below in Fig.1 and Fig.2. It can be concluded that both figures are quite the same.

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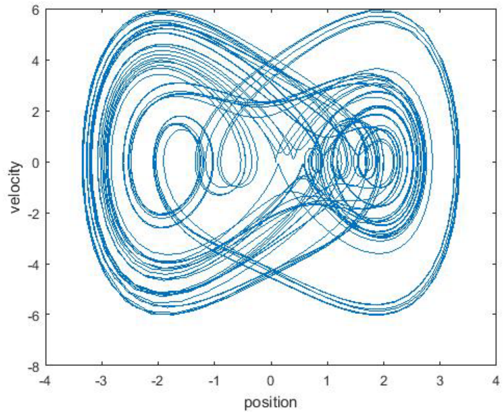

Fig.1. Present test problem solved by the Runge-Kutta 4^{th} order numerical method applied to Duffing equation. The oscillator was started at an initial position x_0 =3.0 and an initial velocity $y_0 = 4.0$ at initial time t_0 ,with damping parameter $b = 0.05$,forcing amplitude $K = 7.5$, frequency $\omega = 1$ and duration of the run $t_d = 200$.

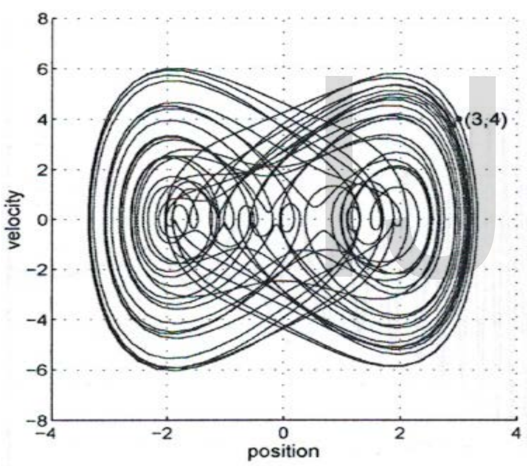

Fig.2. Known test problem solution for the phase portrait of Duffing equation. The parameters are the same as mentioned in the title of the Fig1.

4.Results and discussion In order to achieve the objective specified in the section 2, calculations were performed corresponding to the three accepted Scenarios.

Scenario 1: 3 rd order nonlinearity and damping, no sinusoidal force, no chaos, no shock.

It is assumed that there is 3^{rd} order nonlinearity, damping coefficient b and no sinusoidal time dependent force for the Duffing equation. Also there are no chaos and no shock applied. The results of the calculations are depicted in the Fig.3-Fig.4. They show the system of damped nonlinear oscillations without features characterizing chaos . The envelopes of graphs are decreasing curves. Such graphs are often analyzed in the theory of oscillations.

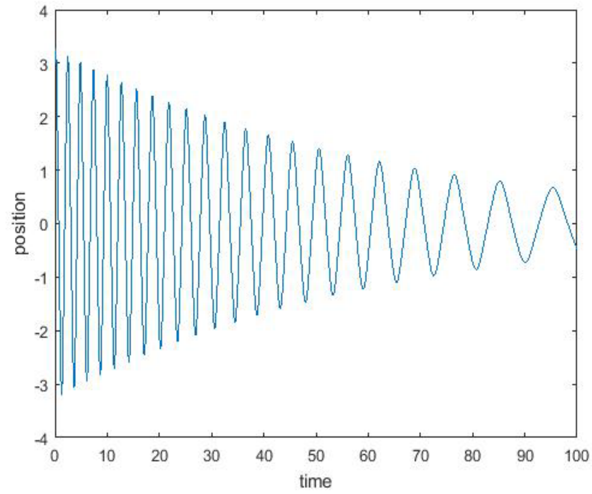

Fig.3. The position x vs time t for the 3^{rd} order, damped unforced Duffing equation with $b = 0.05$ and $K = 0$, $\omega = 0$. For $t_0 = 0$ position $x_0 = 3.0$, duration of the run $t_d = 100$.

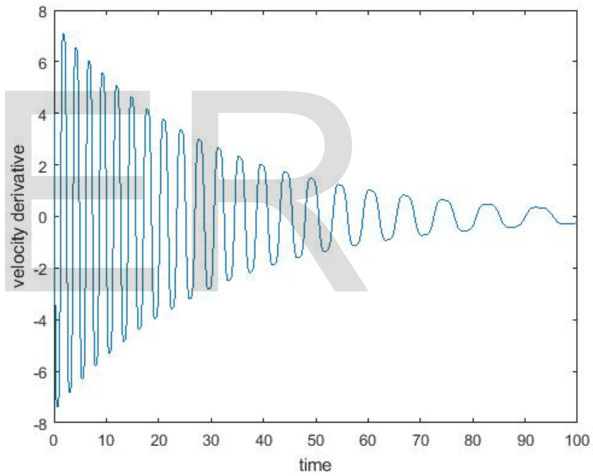

Fig.4. The velocity y vs time t for the 3^{rd} order damped unforced Duffing equation. There are the following parameters: $b = 0.05$, $K = 0$, $\omega = 0$. For $t_0 = 0$ velocity $y_0 = 4.0$, duration of the run is $t_d = 100$.

Scenario 2: 3 rd order nonlinearity and damping, sinusoidal force, chaos,no shock.

The governing parameters are defined in such a way that parameters b, K and ω are non-zero .Chaotic features appear when $50 < t_{chaos} < 100$. The results of calculations are depicted in the Fig.5 and Fig.6 . The peculiarity is that on the Fig.5 and Fig.6 diagrams contain sequential deterministic and chaotic parts. The boundary of the sections refers to the moment of time corresponding to the arrival of the exciting force with the given parameters K, ω .Shock-term is absent.

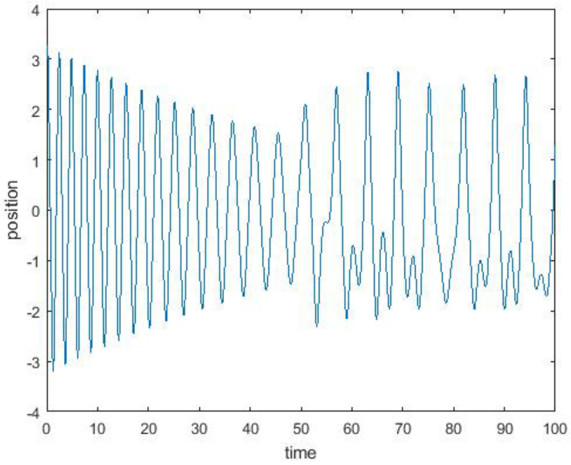

Fig.5. The position x vs time t for the 3^{rd} order nonlinear, damped, forced Duffing equation if $b = 0.05$, $K = 3.75, \omega = 1$; no shock-term. Position $x_0 = 3.0$ at time $t_0 = 0$, duration of the run is $t_d = 100$.

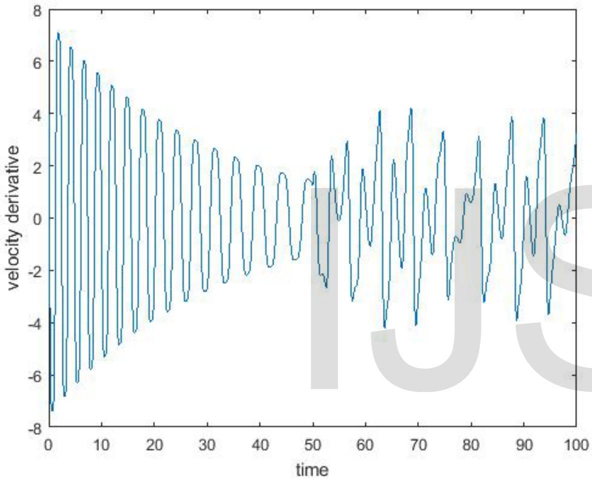

Fig.6. The velocity y vs time t for the 3^{rd} order nonlinear damped forced Duffing equation with $b = 0.05$, $K = 3.75$, $\omega = 1$; no shock-term is applied. Velocity $y_0 = 4.0$ at time $t_0 = 0$, duration of run is $t_d = 100$.

The presence of elements of chaos in solutions significantly reduces the predictability of the obtained solutions and makes it difficult to use them for practical purposes.

Scenario 3: 3^{rd} order non-linearity,damping, sinusoidal force, chaos,applied negative shock.

With regards to this Scenario the governing parameters for the 3^{rd} order nonlinear Duffing oscillator are nonzero b, K and ω. Also negative shock-term $\delta(t)$ was applied using equation (5) where shock parameters are :

$H = 10.146$ -shock intensity,

$t_2 - t_1 = 50$ -shock duration time.

Results of calculations are given in the Fig.7 and Fig.8.

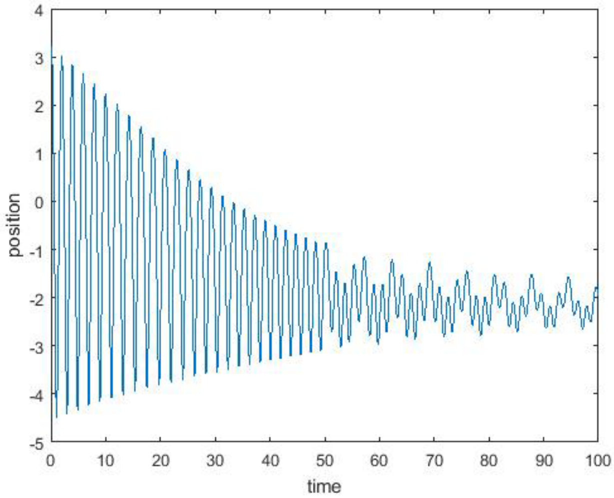

Fig.7. The position x vs time t for the 3^{rd} order nonlinear damped forced Duffing oscillator with $b = 0.05$ and $K = 3.75$, $\omega = 1$; negative shock-term is applied. Position $x_0 = 3.0$ at time $t_0 = 0$, duration of run is $t_d = 100$.

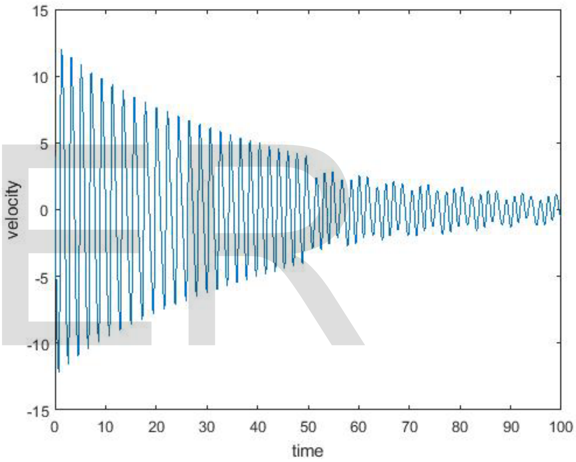

Fig.8. The velocity y vs time t for the 3^{rd} order nonlinear damped forced Duffing oscillator with $b = 0.05$, $K = 3.75$, $\omega = 1$; negative shock-term is applied. Velocity $y_0 = 4.0$ at time $t_0 = 0$, duration of run is $t_d = 100$.

Results of the calculations presented in the Fig.7 and Fig.8 may be compared with similar graphs depicted in the Fig.5 and Fig.6. The peculiarity is that on the Fig.7 and Fig.8 chaotic features are indeed essentially mitigated with comparison to the Fig.5 and Fig.6.

4.Conclusions

It can be concluded from the presented results of the computations that damped, forced nonlinear Duffing equation solutions indeed demonstrate essential mitigation of the chaotic components for both positions and velocities amplitudes due the negative rectangular impact-term with given height and width implemented in the present work.

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